Vector Algebra I

Scalars and Vectors (1)

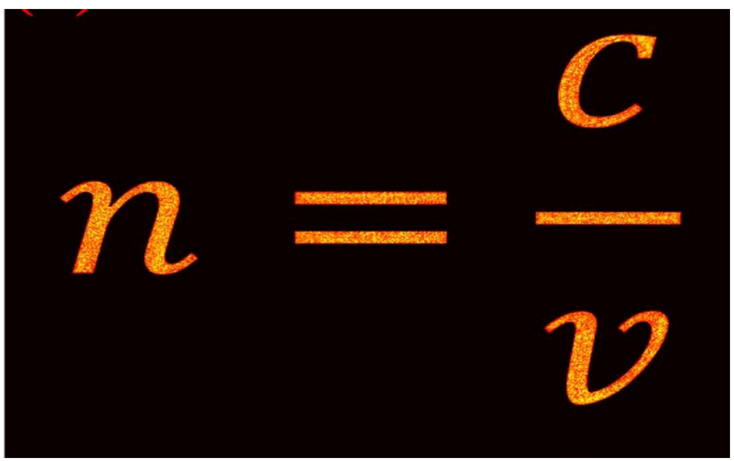
- Vector analysis is a mathematical tool with which EM concepts are most conveniently expressed and comprehended.
- A scalar is a quantity that has only magnitude (time, mass, distance, etc)
- A vector is a quantity that has both magnitude and direction (velocity, force, electric field intensity, etc.

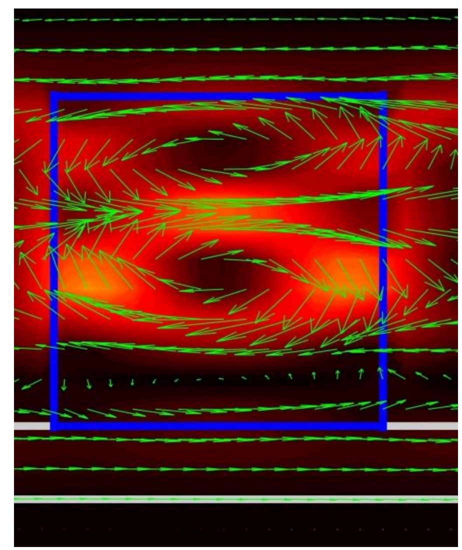
Time

Weight

Scalars and Vectors (2)

- EM theory is essentially a study of some particular fields.
- A field can be scalar or vector and is a function that specifies a particular quantity everywhere in a region.
- Examples of scalar fields are temperature distribution in a building, electric potential in a region.
- The gravitational force on a body in space is an example of vector field.





Scalars and Vectors (3)

- A vector **A** has both magnitude and direction. $\mathbf{A} = \mathbf{a}_A A$
- A unit vector $\mathbf{a_A}$ along **A** is defined as a vector whose magnitude is unity and its direction is along A, that is

$$\boldsymbol{a_A} = \frac{\mathbf{A}}{|\mathbf{A}|}$$

- A vector **A** in Cartesian (or rectangular) coordinates may be represented as

$$\boldsymbol{A} = A_x \boldsymbol{a_x} + A_y \boldsymbol{a_y} + A_z \boldsymbol{a_z}$$

- Where A_x, A_y and A_z are called the components (magnitude) of A in the x, y, and z directions, respectively.
- $\mathbf{a_x}$, $\mathbf{a_y}$ and $\mathbf{a_z}$ are the unit vectors in the x, y, and z directions, respectively.
- Therefore, the unit vector along A may be written as

$$\mathbf{a}_A = \frac{A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Vector Addition and Subtraction (1)

- Two vectors A and B can be added together to give another vector C, that is:

$$\mathbf{C} = \mathbf{A} + \mathbf{B}$$

- The vector addition is carried out component by component
- Thus, if A = (A_x, A_y, A_z) and B = (B_x, B_y, B_z), then

$$\mathbf{C} = (A_x + B_x)\mathbf{a}_x + (A_y + B_y)\mathbf{a}_y + (A_z + B_z)\mathbf{a}_z$$

- Vector subtraction is similarly carried out as:

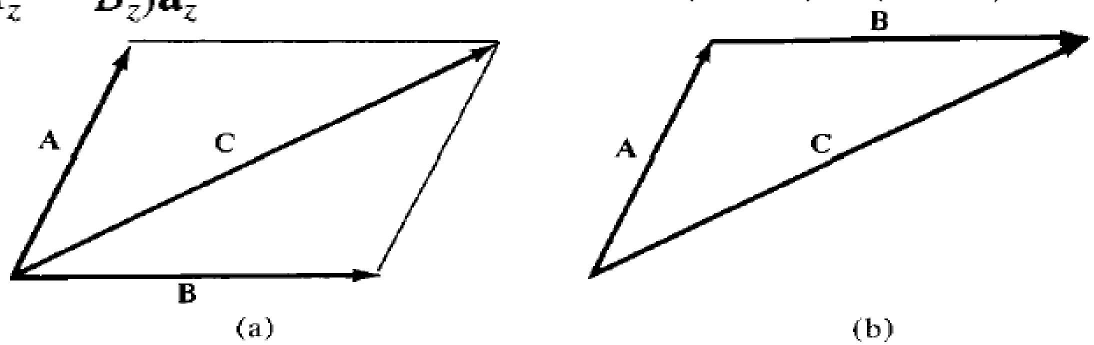
$$\begin{aligned}\mathbf{D} &= \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \\ &= (A_x - B_x)\mathbf{a}_x + (A_y - B_y)\mathbf{a}_y + (A_z - B_z)\mathbf{a}_z\end{aligned}$$

Commutative law: $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.

Associative law: $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$.

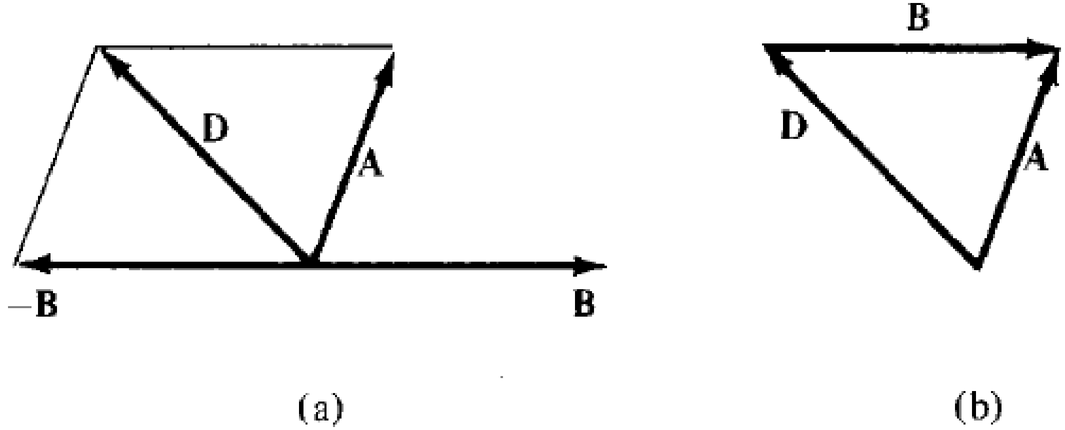
Vector subtraction is neither

1. **Commutative, nor**
2. **Associative.**



Vector addition $\mathbf{C} = \mathbf{A} + \mathbf{B}$: **(a)** parallelogram rule, **(b)** head-to-tail rule.

Vector Addition and Subtraction (2)



Vector subtraction **D** = **A** − **B**: (**a**) parallelogram rule,
(**b**) head-to-tail rule.

- The three basic laws of algebra obeyed by any given vectors A, B, and C, are summarized as follows:

Law	Addition	Multiplication
Commutative	$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$	$k\mathbf{A} = \mathbf{A}k$
Associative	$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$	$k(\ell\mathbf{A}) = (k\ell)\mathbf{A}$
Distributive	$k(\mathbf{A} + \mathbf{B}) = k\mathbf{A} + k\mathbf{B}$	

Vector subtraction is neither

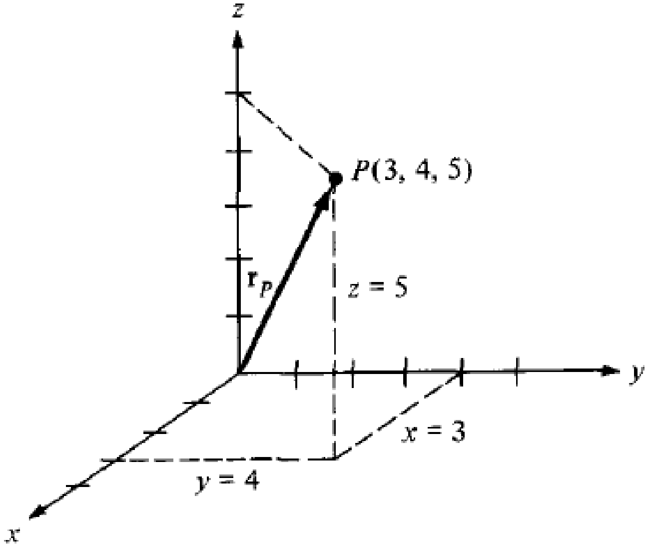
1. **Commutative, nor**
2. **Associative.**

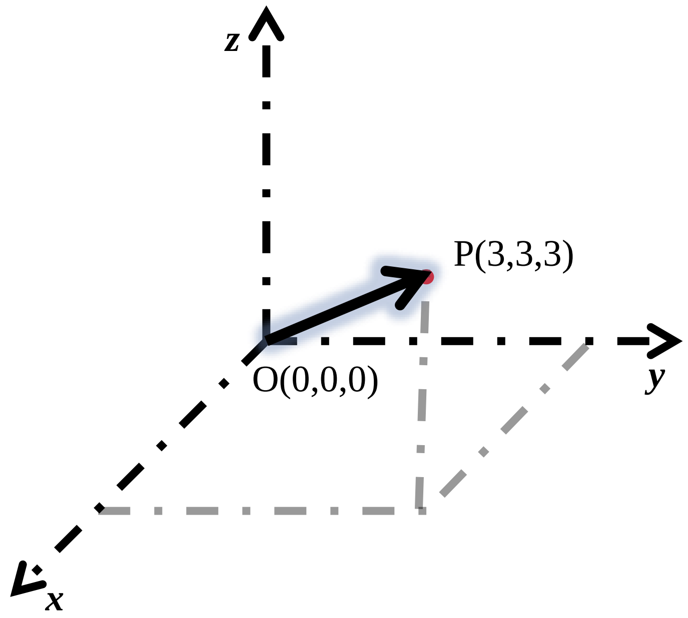
Position Vector

- Point P in Cartesian coordinates may be represented by (x, y, z)
- The position vector r_p (or radius vector) of point P is defined as the directed distance from the origin O to P, i.e.

$$\mathbf{r}_P = OP = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$$

- For point (3, 4, 5), and point (3,3,3) the position vectors are shown in the figures below.

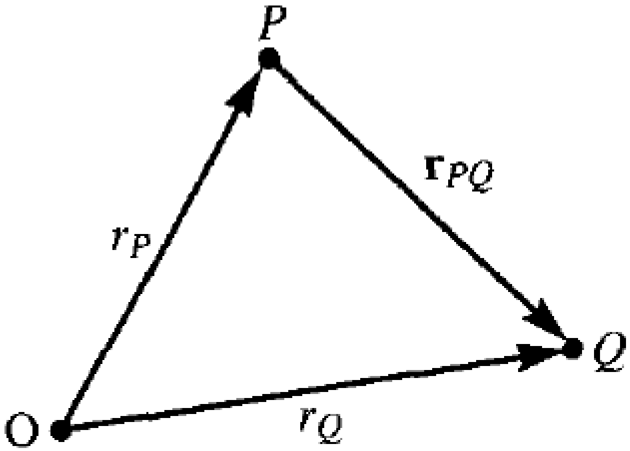




Distance Vector

- The distance vector is the displacement from one point to another
- For two points P and Q given by (x_P, y_P, z_p) and (x_Q, y_Q, z_Q), the distance vector (or separation vector) is the displacement from P to Q, that is.

$$\mathbf{r}_{PQ} = r_Q - r_P$$
$$= (x_Q - x_P)\mathbf{a}_x + (y_Q - y_P)\mathbf{a}_y + (z_Q - z_P)\mathbf{a}_z$$



- Both P and A may be represented in the same manner as (x, y, z) and (A_x, A_y, A_z), respectively.
- However, the point P is not a vector; only its position vector OP is a vector.
- A vector field is said to be constant or uniform if it does not depend on space variables x, y, and z.
- For example, vector B = $3a_x - 2a_y + 10a_z$ is a uniform vector while vector A = $2xya_x + y2a_y - xz2a_z$ is not uniform.

Vector Multiplication – Dot Product

- The dot product of two vectors A and B, written as A • B, is defined geometrically as the product of the magnitudes of A and B and the cosine of the angle between them

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta_{AB}$$

- Also called scalar product because it yields a scalar quantity.
- If A = (A_x, A_y, A_z) and B = (B_x, B_y, B_z), then

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

- Note that

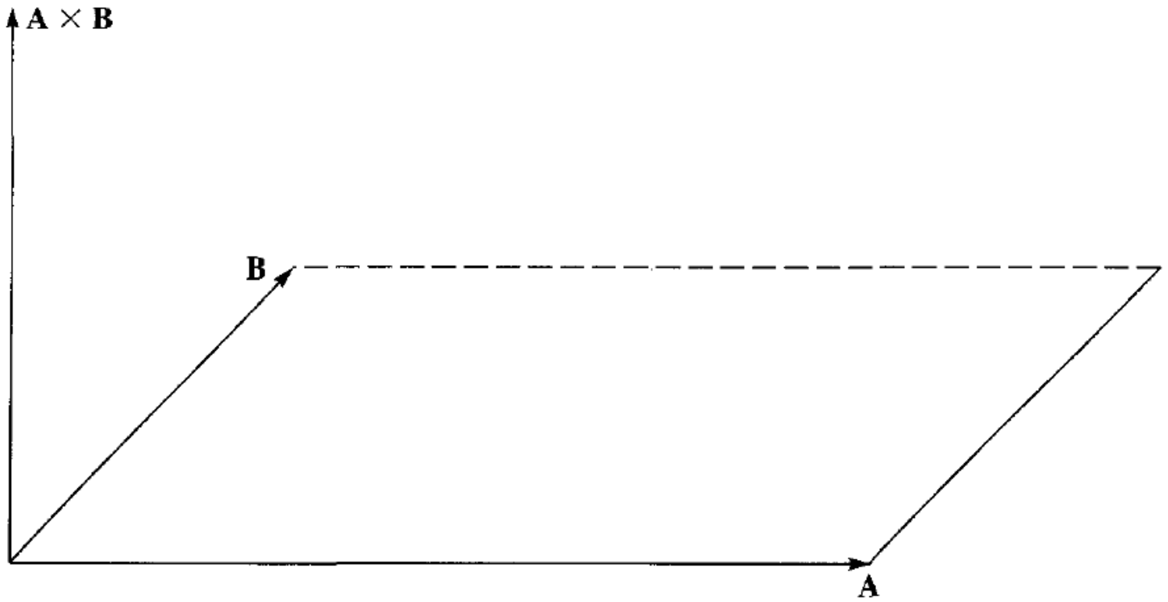
$$\mathbf{a}_x \cdot \mathbf{a}_y = \mathbf{a}_y \cdot \mathbf{a}_z = \mathbf{a}_z \cdot \mathbf{a}_x = 0$$

$$\mathbf{a}_x \cdot \mathbf{a}_x = \mathbf{a}_y \cdot \mathbf{a}_y = \mathbf{a}_z \cdot \mathbf{a}_z = 1$$

Vector Multiplication – Cross Product (1)

- The cross product of two vectors A and B, written as A x B, is a vector quantity whose magnitude is the area of the parallelepiped formed by A and B and is in the direction of the right thumb when the fingers of the right hand rotate from A to B.

$$\mathbf{A} \times \mathbf{B} = AB \sin \theta_{AB} \mathbf{a}_n$$



Vector Multiplication – Cross Product (2)

- If $A = (A_x, A_y, A_z)$ and $B = (B_x, B_y, B_z)$, then

$$\boxed{\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}}$$

$$= (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_x)\mathbf{a}_z$$

- Note that:

$$\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$

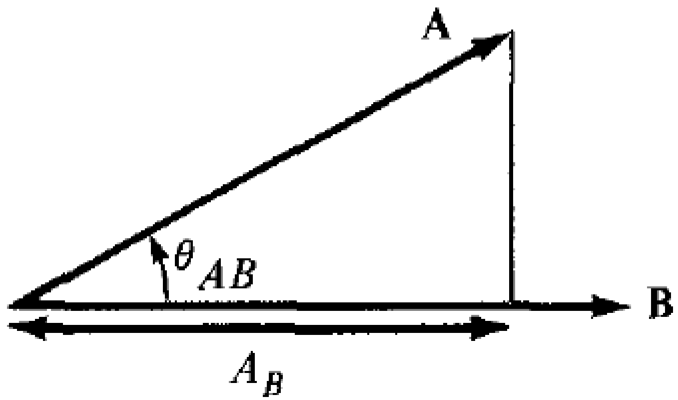
$$\mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x$$

$$\mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y$$

Components of a Vector

- Scalar Component:

$$A_B = \mathbf{A} \cdot \mathbf{a}_B$$



- Vector Component:

$$\mathbf{A}_B = A_B \mathbf{a}_B = (\mathbf{A} \cdot \mathbf{a}_B)\mathbf{a}_B$$